

## <sup>2</sup> ghYffY]WX 'd`Ubh'i a ghf]hhYbY , VYfk UWXi b[ 'j cb 'A YggYb[ Yf!8]YbghYb°

 $^2$  ghYffY]WX 'ghY\h'j cf'Y]bYa 'Zc`[YbfY]WXYb'GWXf]hh']b'XYf , VYfk UWNib['X][]hU`Yf'?caaib]\_Uh]cbga]hhY`"'=a 'FU\aYb XYg'FY[]Yfi b[qdfc[fUa a q'd`Ubh'\byba]b]qhYf'; Yf\UfX ?UfbYfžX]Y, VYfk UWXi b[ j cb A YggYb[ Yf!8]YbghYb Y]bni ZØ\fYbž'i a '[Yn]Y`h']b'9]bnY`ZÀ``Yb'[Y[Yb'dchYbn]Y``Y GhfuZhÀhYfjcfni[Y\Yb"?UfbYfVYhcbhžXUggYgg]WXb]WXh ia Y]bYZÀWXYbXYW\_YbXY, VYfk UWXib[~\UbXY`hž:gcbXYfb V]g'' \$'9]bnY`ZÀ``Yb'XifW\'XYb'JYfZUgqib[ggW\ihn"'CVkc\` Y]bY'bYi Y'J YfZUggi b[gVYgh]a a i b['b]W\h'nk]b[YbX  $YfZcfXYf^{W}^{-}W^{-}ghz^{-}k^{-}fX^{-}Y^{-}bY^{-}VfY^{-}hY^{-}dc^{-}h^{-}gW^{-}Y^{-}A^{-}Y^{-}h^{-}ZØf^{-}XUg^{-}y^{-}h^{-}y^{-}h^{-}ZØf^{-}XUg^{-}y^{-}h^{-}Y^{-}y^{-}h^{-}ZØf^{-}XUg^{-}y^{-}h^{-}ZWg^{-}YUg^{-}y^{-}h^{-}ZWg^{-}y^{-}h^{-}ZWg^{-}y^{-}y^{-}h^{-}ZWg^{-}y^{-}y^{-}y^{-}y^{-}y^{-}$ Jcf\UVYb'U`g'k ØbgW\Ybgk Yfh'YfUW\hYh"'8]Yg'VYf]W\hYb ?cga c"'Ni gÀhn`]WX'\UVYb'X]Y'Gcn]U`XYa c\_fUh]gWXY'DUfhY] <sup>2</sup> ghYffY]WXg'fGD<sup>2</sup> Ł'i bX'X]Y BYcg'5ba Yf\_i b[Yb'ni 'XYa ; YgYhnYgYbhkifZ'Y]b[YfY]WXh''?UfbYf'd`Ubhž'X]YgYJcfgWX`À[Yijcf/Uad∧Y``]d/

2 ghYffY]WX 'ghY\h'j cf'Y]bYa 'Zc`[YbfY]WXYb'GWXf]hh']b'XYf
, VYfk UWXi b['X][]hU'Yf'?ca a i b]\_Uh]cbga ]hhY`"-a 'F U\a Yb'XYg
FY[]Yfi b[gdfc[fUa a g'd`Ubh'+bbYba ]b]ghYf'; Yf\UfX'?UfbYfžX]Y
, VYfk UWXi b['j cb'A YggYb[Yf!8]YbghYb'Y]bni ZØ\fYbži a '[Yn]Y`h
]b'9]bnY`ZÀ``Yb'[Y[Yb'dchYbn]Y``Y'GhfUZhAhYf'j cfni [Y\Yb"'?UfbYf
VYhcbhžXUgg'Yg'g]WX'b]WXhi a 'Y]bY'ZÀWXYbXYW\_YbXY
, VYfk UWXi b['\UbXY`hžgcbXYfb'j]Y`a Y\f'i a 'X]Y'6YUfVY]hi b[
j cb'A\f`]WX'[YgWXAhnhYb'&) 'V]g'' \$'9]bnY`ZÀ``Yb'Xi fWX'XYb
JYfZUggi b[ggWXi hn"'CVk c\``Y]bY'bYi Y'JYfZUggi b[gVYgh]a a i b[
b]WXh'nk]b[YbX'YfZcfXYf`]WX']ghž'k]fX'Y]bY'VfY]hY'dc`]h]gWXY
A Y\f\Y]h'ZØf'XUg'J cf\UVYb'U`g'k ØbgWXYbgk Yfh'YfUWXhYh"8]Yg
VYf]WXhYb'?cga c"

 $\label{lem:linear_norm} Ni\ g\ h\ `]\ W\ `\ U\ Y\ b\ `X]\ Y\ `Gcn]\ U\ `X\ Y\ a\ c_fU\ h]\ g\ W\ Y\ `DU\ f\ h\ Y\ ]\ '\ 2\ g\ h\ Y\ f\ f\ Y\ ]\ W\ g$ 

fGD<sup>2</sup> Łi bX'X]Y'BYcg'5ba Yf\_i b[Yb'ni 'XYa '; YgYhnYgYbhk i fZ Y]b[YfY]W\h'''?UfbYf'd`Ubhž'X]YgY'JcfgW\`\A[Y'jcf'XYf'I a gYhni b[ XYg'; YgYhnYg'Y]bni UfVY]hYbž'k Ug'X]Y'dc`]h]gW\Y'8]g\_i gg]cb'fi bX i a 'XUg'H\Ya U'k Y]hYf'Ub\Y]nh"

## : Ua ]`]YbbUW\ni [ 'i bX'A ][ fUh]cb

DUfU``Y``ni f'8]g\_i gg]cb'i a 'X]Y'X][]hU`Y', VYfk UW\i b['d`Ubh ? UfbYf'Ui W\ Y]bY'BYi fY[Y`i b['XYg': Ua]`]YbbUW\ni [g'Xi fW\ Y]bY Ei chYbfY[Y`i b["8]YgY'; YgYhnYgÀbXYfi b['n]Y`h'XUfUi Z'UVž'XYb : Ua]`]YbbUW\ni ['j cfØVYf[Y\YbX'ni 'ghcddYb'i bX'Ui gni gYhnYb" 2 ghYffY]W\ \Uh'X]Y'9I!? ca a ]gg]cb'ØVYf'X]YgY'9bhgW\Y]Xi b[']b ?Ybbhb]g'[YgYhnh'i bX'd`Ubh'X]Y'9]bZØ\fi b['Y]bYg ?cbh]b[Ybh]Yfi b[ ga cXY``g 'ZØf'fY[]cbU`'j YfhfÀ[`]W\Yb'Ni ni ["8UVY]'gc``Yb'VYgh]a a hY'?f]hYf]Yb'k]Y'GdfUW\\_Ybbhb]ggYž GY`VghYf\U`hi b[gZÀ\][\_Y]h'i bX'j cf\UbXYbY'GW\i `d`ÀhnY a U\[YV`]W\ 'gY]b"

## ?ifnY`K YfVYY]bV`YbXib[

9]b'k Y]hYfYg'nYbhfU`Yg'H\Ya U'V`Y]Vh'X]Y'FØW\_ZØ\fi b['j cb 6Øf[Yfb'Ui g'?f]gYbfY[]cbYb'k]Y'Gmf]Yb'i bX'5Z[\Ub]ghUb"'8Yf c\_i g'`]Y[h'\]YfVY]'Ui Z'XYf'FØW\_ZØ\fi b['j cb'; YZÀ\fXYfb'i bX GhfUZhÀhYfbž'cVk c\``XYfnY]h'\_Y]bY'\_cb\_fYhYb'D`ÀbY'ZØf FØW\_Y\fnYbhfYb'VYghY\Yb"

## $8 \hspace{0.1cm}][\hspace{0.1cm}] \hspace{0.1cm} \text{hU`Y`FYW\& hY`i bX`Yh\& ]} \hspace{0.1cm} gW\&Y`,\hspace{0.1cm} VYf`Y[\hspace{0.1cm}|\hspace{0.1cm}i\hspace{0.1cm}b[\hspace{0.1cm}] Yb$

a '?cbhYl h'XYf'[ Yd`UbhYb'A U» bU\a Yb']b'2 ghYffY]W\'ghY\Yb'Ui W\
X]Y'Df]bn]d]Yb'XYf'X][ ]hU`Yb'HfUbgZcfa Uh]cbž'X]Y'Ui Z'XYf'9I !9VYbY
ZYgh[ Y`Y[ h'k i fXYbž']a ': c\_i g"'; Ya À» 'XYf'8][ ]hU`'GhfUhY[ m'XYf
9I 'gc``Yb'A YbgW\Yb'i bX']\fY'FYW\hY']a a Yf']b'XYb'A ]hhY`di b\_h
X][ ]hU`Yf'9bhk ]W\_`i b[ Yb'[ YghY``h'k YfXYb"'8]Yg'i a ZUggh'5gdY\_hY
k ]Y'XYb'i b]j YfgY``Yb'Ni [ Ub[ 'ni 'HYW\bc`c[ ]Ybž'X]Y
; Yk À\f`Y]ghi b[ 'j cb'K U\`ZfY]\Y]h']a '\DhYfbYh'i bX'X]Y'G]W\Yf\Y]h
gck ]Y': ÒfXYfi b[ 'j cb'\D\_`i g]cb'i bX'Gc`]XUf]hÀh']b'XYf'X][ ]hU`Yb
K Y`h"

8]Y'9I 'j YfdZ]W\hYh'g]W\žY]bYb'X][]hU`Yb'K UbXY``ni '[YghU`hYbž'XYf XUg'@YVYb'U``Yf'j YfVYggYfh'i bX'XYb'Ni [Ub['ni 'ZU]fYb'X][]hU`Yb 6YX]b[i b[Yb'[UfUbh]Yfh''8]YgY'; fi bXg\hnY'g]bX'Ui W\'Z\Øf'X]Y 8]g\_i gg]cb'\@VYf', VYfk UW\i b[ga U\s\bU\a Yb'j cb'6YXYi hi b[\ž'XU g]Y'X]Y'J YfUbhk cfhi b['XYf'Dc']h]\_'i bhYfghfY]W\Yb\z'X]Y ; fi bXfYW\hY'i bX'X]Y'Df]j Uhgd\\hatafY'XYf'6\Øf[Yf'ni 'k U\fYb''

Cfh'XYg'; YgWXY\Ybg8YhU]`g'ni f'A Y`Xi b[; YbUi Yf'Cfh VY\_Ubbh3<sup>2</sup> ghYffY]WX6YghY'FYZYfYbn\_cga c"Uh

Gci fWY. X ]Y!bUWX f]WX hYb"Uh

8 YhU]`g

6Ygi WXYb G]Y i bg Ui Z b!U[ "XY