

I b]cb'6Yf`]b.'<c``YfVUW\'hfÀi a h'j cb'8: 6! 6Yfi Zi b['bUW\'?`UggYbYf\U`h°

I b]cb'6Yf`]b'g]W\Yfh'a]h'Y]bYa '(.('[Y[Yb'Ghi hh[Ufh'XYb ?`UggYbYf\U`h"'<c``YfVUW\'\c\ZZh'Ui Z BUh]cbU`a UbbgW\UZhgVYfi Zi b["

7\Uf`chhYbVi f[!K]`a YfgXcfZž'8Yi hgWX`UbX'!'I b]cb'6Yf`]b\Uh'Ua '&\$"'5df]`'&\$&) 'Y]b'XfUa Uh]gWXYg'(.(!I bYbhgWX]YXYb
[Y[Yb'XYb'JZ6'Ghi hh[Ufh'YffY]WXh'i bX'g]WX'XUa]h'j cfnY]h]['XYb
?`UggYbYf\U`h']b'XYf'6i bXYg`][U'[Yg]WXYfh"'8]Y'BUWXf]WXh']gh
VYgcbXYfg'YfZfYi `]WX'ZØf'X]Y'5b\Ab[Yf'XYg'JYfY]bgž'XU
[`Y]WXnY]h]['<Y]XYb\Y]a '[Y[Yb'6UmYfb'A ØbWXYb'j Yf`cfž'gcXUgg
I b]cb'6Yf`]b'b]WXh'Ui Z'FUb['%) 'ni f@W_ZU``Yb'_Ubb"'8]YgY'GU]gcb
j Yf`]YZ'YXcWX'UbXYfg'U`g'Yfk UfhYh'ZØf'X]Y'6Yf`]bYfž'X]Y'hfchn
UbZAb[`]WXYf'GWXk]Yf][_Y]hYb'bib b'X]Y'GhUV]`]hAh'ni f@W_[Yk]bbYb"

6YbYX]Wh< c``YfVUW\žXYf'HcdgWcfYf'j cb'l b]cb'6Yf`]bž\Uhig]W\
Xi fW\gY]bY'VYY]bXfi W_YbXYb'@Y]ghi b[Yb']b'X]YgYf'GU]gcb
\Yfj cf[YhUb"BUW\gY]bYa'K YW\gY`]a 'Gca a Yf'&\$&' 'j cb
K Y\Yb'K]YgVUXYb'\UhiYf'g]W\gW\bY``ni a ': Ub`]YV`]b[
Ybhk]W_Y`h"'\d\X]YgYf'GU]gcb'Yfn]Y`hY'<c``YfVUW\VYfY]hg'UW\h'HcfYž
ni 'XYb'<Ò\Ydi b_hYb'nÀ\`h'gY]b'YbhgW\Y]XYbXYf'HfYZZYf'[Y[Yb
XYb': 7'6UmYfb']a 'A Àfnž'XYf'a U»[YV`]W\ni a 'ZfØ\nY]h][Yb
?`UggYbYf\U`h'VY]hfi ["'GY]bY': cfa 'nY][hž'XUgg'Yf'Ui W\'U`g
9]bk YW\gY`qd]Y`Yf'[i hY'@Y]qhi b[Yb'YfVf]b[Yb'_Ubb"

< c``YfVUWXg'BUh]cbU`a UbbgWXUZhghfÀi a Y

BUh]cbU`a UbbgW\UZh'VYfi ZYb'ni 'k YfXYbž'k Ug'ZØf']\b'XYb\Y]`][Yb'; fU``]a ': i » VU`` 'XUfghY``Yb'k ØfXY"' \D'Y]bYa '; YgdfÀW\jcf'XYa 'Gd]Y`'[Y[Yb'Ghi hh[Ufh'nY][hY'Yf'g]W\'ni j Yfg]W\'h`]W\']b\6Yni ['Ui Z'gY]bY'Ei U`]hÀhYb'i bX'7\UbW'b'Ui Z'Y]bY'Bca]b]Yfi b[ž]bgVYgcbXYfY'XU'6i bXYghfU]bYf'\J']Ub'BU[Y`ga Ubb']b'Y]b][YbKcW\Yb'XYb'?UXYf'ZØf'X]Y'BUh]cbg'@YU[i Y!Gd]Y`Y'VY_Ubbh[YVYbk]fX"

8]Y'VYj cfghY\YbXYb'@ÀbXYfgd]Y`Y'g]bX'ZØf'X]Y
BUh]cbU`a UbbgW\UZh'j cb'[fc»Yf'6YXYi hi b["'5a '&\$"'A Àfn'&\$&)
VY[]bbh'XUg'@ÀbXYfgd]Y`'U\f'a]h'Y]bYa 'J]YfhY`Z]bU`gd]Y`']b'XYf
BUh]cbg'@YU[iY'[Y[Yb'\=hU`]Yb''<c``YfVUW\'_ObbhY'Y]bY'[i hY
9f[Àbni b['ZØf'Y]b'HYUa 'gY]bž'XUg'a]h'HU`YbhYb'k]Y': `cf]Ub'K]fhn
i bX'\Ja U`'A i g]U`U'[Ygd]\W_h']ghž'i bX'XUg'ni XYa 'X]Y'<c\ZZbi b['Ui ZbYi Y'9fZc`[Y'ZØf'X]Y'K A '&\$&*`qW\Øfh"

9]b 6`]W_ 'Ui Z'X]Y 'J YfY]bg[YgW]W hY

8]Y'U_hi Y``Y'GU]gcb']gh'ZØf'I b]cb'6Yf`]b'Y]b'k Y[k Y]gYbXYf
A ca Ybh''C Vk c\``XYf'K Y['Xcfh\]b'hi fVi `Ybh'k Ufž'_Ubb'g]W\`X]Y
A UbbgW\UZh'Ui Z'Y]bY'GYf]Y'j cb'gYW\g'i b[YgW\`U[YbYb'Gd]Y`Yb
j Yf`UggYbž'X]Y'XUg'GY`Vghj YfhfUi Yb'i bX'X]Y'HYUa gh]a a i b[
Yf\YV`]W\'ghY][Yfb'''<c``YfVUW\'gd]Y`hY'Ui W\']b'XYf
I %, !BUh]cbU`a UbbgW\UZhž'k c'Yf']b'nk Y]'9]bgAhnYb'Y]bYb'HfYZZYf
Yfn]Y`hY'''Gc``hY'Yg']\a'[Y`]b[Ybž']b'X]Y'5!BUh]cbU`a UbbgW\UZh
VYfi ZYb'ni 'k YfXYbž'k AfY'X]Yg'ZØf']\b'Y]b'7ca YVUW_'Ui Z'XYf
[fc>Yb'6Ø\bY''

±g[YgUa h'nY][h'g]W\ž'XUgg'gck c\``XYf'JYfY]b'U`g'Ui W\`<c``YfVUW\VYfY]h'g]bXž'g]W\'XYb'<YfUi gZcfXYfi b[Yb'XYf'Ni _i bZh'ni 'ghY``Ybi bX'Ui Z'XYb'9fZc`['ni '\cZZYb"'8]Y'bÀW\ghYb'K cW\Yb'k YfXYb

 $\label{thm:linear} YbhgWXY]XYbX`gY]b\check{z}`gckc\``ZØf`<c``YfVUWXg`]bX]j~]Xi~Y``Y`N]Y`Y`U`g~Ui~WX~ZØf`X]Y`A~UbbgWXUZh`]bg[~YgUa~h"$

I b]cb'6Yf`]b'\Uh'Xi fW\'XUg'I bYbhgW\]YXYb'[Y[Yb'Ghi hh[Ufh'i bXX]Y'XUa]h'g]W\Yf[YghY``hY'6i bXYg`][U!Ni [Y\Òf][_Y]h'Y]bY'dcg]h]j Y 6Ug]g'[YgW\UZZYbž'Ui Z'XYf']b'XYb'Zc`[YbXYb'GU]gcbg'Ui Z[YVUi h k YfXYb'_Ubb"

8 YhU]`g	
Cfh	7\Uf`chhYbVif[!K]`aYfgXcfZž'8YihgW\`UbX
EiY``Yb	• k k k "VYf`]b!`]j Y"XY
	• kkk"giYXXYihgWXY"XY
	XY "ghUh]ghU"Wc a

6Ygi WXYb G]Y i bg Ui Z b!U["XY