

; Y`i b[YbY'JYfa]gghYbgi WXY.'9Xk]b'@"']b : i `XU'k c\`VY\U`hYb'Ui Z[YZi bXYb

9fZc`[fY]WXY'JYfa]gghYbgi WXY']b':i`XU.'9Xk]b'@''kifXYkc\`VY\U`hYb'[YZibXYb"'8Ub_'Ub'X]Y'6Øf[Yf'ZØf']\fYlbhYfghØhnib[°

: i `XU` '♣ 'Y]bYa 'YfZfYi `]WXYb'K YbXYdi b_h'_cbbhYb'X]Y
G]WXYf\Y]hg_fÀZhY'Ua ': fY]hU[UVYbX'Y]bYb'j Yfa]gghYb'DUh]YbhYb
Z]bXYbž'XYf'ni j cf'Ui g'Y]bYa '?fUb_Yb\Ui g'Ybh`Ui ZYb'k Uf"'9Xk]b
@'ž'XYggYb'UV[Àb[][Yf'Ni ghUbX'VYfY]hg'Ua ': fY]hU[ž'XYa '&' "
5i [i ghž'ZYgh[YghY``h'k i fXYž'_cbbhY'bUWX'i a ZUb[fY]WXYb
Gi WXU_h]cbYb'i bX'Y]bYa '5i Zfi Z'Ub'X]Y'² ZZYbh`]WX_Y]h'[Y[Yb'&%
I \f'k c\`VY\U`hYb'Ui Z[YZ bXYb'k YfXYb"'Gc`WXY'J cfZÀ``Y'fi ZYb'cZh
[fc»Y'6Ygcf[b]g'\Yfj cfž']bgVYgcbXYfY'VY]'XYb'5b[Y\Òf][Yb'XYg
6YhfcZYYbYb"

8]Y'G]hi Uh]cb'YfZcfXYfhY'gW\bY``Yg'<UbXY`bž'k Yg\U`V'XUg
Dc`]nY]dfÀg]X]i a 'Cgh\YggYb'gcZcfh'a Y\fYfY'GhfY]ZYb'j YfgUbXhY"
8]Y'6YUa hYb'Xi fW\gi W\hYb'Y]b[Y\YbX'X]Y'bU\Yb': fY]ZÀW\Yb
fi bX'i a 'XUg'?fUb_Yb\Ui g'i bX'gYhnhYb'ni gÀhn`]W\'Y]bYb
Dc`]nY]\i VgW\fUi VYf'Y]bž'i a 'j cb'cVYb'Y]bYb'VYggYfYb', VYfV`]W_
ni 'Yf\U`hYb"'Gc`W\Y'A U» bU\a Yb'g]bX'b]W\h'bi f'k]W\h][ži a 'X]Y
G]W\Yf\Y]h'XYf'j Yfa]gghYb'DYfgcb'ni '[Yk À\f`Y]ghYbž'gcbXYfb
Ui W\'i a 'X]Y'Gcf[Yb'XYf'5b[Y\Òf][Yb'gW\bY``ni 'nYfghfYi Yb"

2 ZZY b h`]WX _ Y]hg ZU \ b X i b [nY][h K]f_i b [

 $8]Y^2 ZZYbh`]WX_Y]hgZU\bXib[`]gh`Y]b`k]f_ib[gjc``Yg`A]hhY``]b`XYf\\ JYfa]gghYbgi WXY"8ifWX`X]Y`fYWXhnY]h][Y`JYfVfY]hib[`jcb \div Zcfa Uh]cbYb`YffY]WXYb`X]Y`6Y\ÒfXYb`Y]b`VfY]hYg`DiV`]_iažibX cZh`_caaYb`<]bkY]gY`jcb`VYgcf[hYb`6Øf[Yfbž`X]Y$

a Ò[`]WXYfk Y]gY Yhk UgʻG]WX hVUfYgʻVY]hfU[Ybʻ_ÒbbYb"'±bʻX]YgYa : U```gWX]YbʻXYfʻ9]bgUhnʻXYfʻDc`]nY]ʻ]bʻ?ca V]bUh]cbʻa]hʻXYf I bhYfghØhni b[ˈXYfʻ6Yj Ò`_Yfi b[ˈYfZc`[fY]WX [Yk YgYbʻni ˈgY]bžk Ug j]Y`Yʻ]bʻXYfʻFY[]cbʻdcg]h]jʻk U\fbU\a Yb"

 $8]Y \cdot Dc \cdot]nY] \cdot gY \cdot Vgh \cdot nY][hY \cdot g]WX \cdot XUb _VUf \cdot ZØf \cdot X]Y \cdot A]h \setminus] \cdot ZY \cdot XYf \cdot 6 Øf[Yf'' \pm b' \cdot] \setminus fYf \cdot A]hhY] \cdot i b [\cdot \setminus c \lor Yb \cdot g]Y \cdot \setminus Yf \cdot c f \check{z} \cdot k]Y \cdot k]WX \cdot h][\cdot X]Y$ Ni gUa a YbUf \text{VY} \text{h'nk}]gWX \text{Yb' \text{XYb' \text{G}}WX \text{Yf} \text{Y}]hg_f \text{A} \text{ZhYb' \text{i}} \text{i} \text{b' \text{XYf}} \ N]j]\text{VYj} \text{O`_Yfi b}[\cdot ZØf \text{XYb' \text{Yf} \text{Zc}`[\text{fY}]WX \text{Yb' \text{Y}f' \text{Ui} \text{Z'XYf' \text{Gi} WX \text{Y'}]gh'' \text{9g} \text{j' \text{XYf' \text{VY}} \text{hfU}[\text{Yb' \text{_Ubb \text{Z'i}} \text{a} \text{_[YZ\text{A}\fXYhY } \ DYfgcb \text{Yb' \text{g}ha \text{O}[\cdot]WX \text{ni} \text{_Z}b \text{XYb' i} \text{bX'}]b' \text{G}]WX \text{Yf\text{Y}}h' \text{ni} \text{Vf} \text{Ib' \text{Ib}} \text{Vf} \text{Vf} \text{Vf} \text{Ib} \text{If} \text{Ib} \text{Yf} \text{Vf} \text{If} \text{If}

9Xk]b@"kifXY'b]WhbiffYWhnY]h][[YZibXYbžgcbXYfb'kUfUiW\kc\`VY\U`hYb"Gc`W\Y'[`ØW_`]W\Yb'K YbXib[Yb'g]bX']b'XYf\Y_h]gW\Yb'K Y`hž']b'XYf'k]f'`YVYbž']a a Yf'Y]b'; fibX'nif 9f`Y]W\hYfib["'=b'Bch'U[Ybž'k]Y'g]Y'cZh'VY]'j Yfa]gghYb'DYfgcbYbjcf_caaYbž']gh'XYf'gW\bY``Y'=bZcfaUh]cbgUighUigW\nk]gW\YbXYf'Dc`]nY]'ibX'XYf'6YjÒ`_Yfib['YbhgW\Y]XYbX"'9g'ghY][Yfhg][b]Z]_Ubh'X]Y'7\UbW'bž'jYfa]gghY'AYbgW\Yb'fUgW\ni'Z]bXYb"

6YXYi hi b['XYf'gW\ bY``Yb'FYU_h]cb

8]Y '9fY][b]ggY ']b': i `XU'j YfXYi h`]W\Yb'XUg'6YXØfZb]g'bUW\
gW\bY``Ya '< UbXY`b']b'?f]gYbg]hi Uh]cbYb"'J Yfa]gghY'DYfgcbYb
_\ObbYb']b'Y]bYf'J]Y`nU\``j cb'l a gh\\bXYb'j YfgW\k]bXYb\\\zi bX'X]Y
NY]h'gd]Y`h']b'X]YgYb': \\A``Yb'Y]bY'YbhgW\Y]XYbXY'Fc``Y'''\\A'`a Y\f
NY]h'j Yf[Y\h\\\ziXYghc'[f\Os\Yf'k]fX'XUg'F]g]_c\\\ziXUgg'Yhk Ug
GW\\`]a a Yg'dUgg]Yfh''8U\Yf']gh'Yg'k]W\\h][\\\\\\ziXUgg'gck c\\``X]Y'Dc\\]nY]
U`g'Ui W\'X]Y'6\\\\fof[Yf'gW\\bY\``i bX'YbhgW\\`cggYb'\UbXY\`b"

8]Y Ni gUa a YbUfVY]hink]gWXYb XYb [`Y]WXYa '6]`XYfb j cb : fYi bX`]WX_Y]hii bX '9b[U[Ya Ybh]bbYf\U`V 'XYf'; Ya Y]bgWXUZh nY][hz'k]Y 'ni gUa a Yb 'Y]bY '@Ògi b['[YZ bXYb k YfXYb '_Ubb" K Ybb 6Øf[Yf 'VYfY]hig]bXz'ni '\Y`ZYb ii bX '\dangle Zcfa Uh]cbYb 'ni 'hY]`Ybz'_Ubb X]Y 'Dc`]nY] 'YZZY_h]j 'Ui Z']\fY 'A UbXUhY 'fYU[]YfYb ii bX 'k]WXh][Y\dangle Zcfa Uh]cbYb 'ni '[Yk]bbYb" 'Gc`WXY 'GmbYf[]Yb 'g]bX 'XYf 'GWX 'ØggY`ni a '9fZc`[z'VYgcbXYfg'VY] 'XYf 'Gi WXY 'bUWX 'j Yfa]gghYb 'DYfgcbYb"

Ni gUa a YbZUggYbX``Àggh'g]W\`gU[Ybž'XUgg'X]Y`Gi W\Y`bUW\`9Xk]b@''b]W\h'bi f'Y]bY': fU[Y'j cb'Dc`]nY]'i bX'JYfa]gghYb']ghž'gcbXYfbY]b'; Ya Y]bgW\UZhgUbgUhnž'XYf'nY][hž'k]Y'k]W\h]['Yg']ghž'Y]bUbXYfni i bhYfghØhnYb"'8 Ug'9f`YVb]g']b': i `XU'_ÒbbhY'U`g'A cXY```X]YbYbžk]Y'YZZY_h]j Y`Gi W\Y'i bX'dfÀj Ybh]j Y`A U» bU\a Yb'ni f'G]W\Yf\Y]h j cb'9]bnY`dYfqcbYb'cf[Ub]q]Yfh'k YfXYb'_ÒbbYb"

8]Y 'YfZc`[fY]WXY 'FØW__Y\f'jcb'9Xk]b'@'']gh'Y]b'@]WXhV`]W_'ibX'Y]b6YkY]gž'XUgg'9b[U[YaYbh'ibX'?ccdYfUh]cb'jcb'6Øf[Yfb'ibXDc`]nY]'Y]bY'dcg]h]jY'K]f_ib['Yfn]Y`Yb'_ObbYb'''Gc`WXY'JcfZÀ``Yg]bX'YfbghY'9f]bbYfib[Yb'Ub'X]Y':fU[]`]hÀh'XYf'aYbgWX`]WXYbG]WXYf\Y]h'ibX'X]Y'BchkYbX][_Y]h'Y]bYf'kUWXgUaYb'ibXibhYfghØhnYbXYb';YaY]bqWXUZh''

<]bhYf[fi bX]bZcfa Uh]cbYb ni f J Yfa]gghYbgi WX Y</pre>

 $\label{thm:condition} $$JYfa]gghYbgi WXYb^k]Y^X]Y^]b^: i `XU^g]bX^cZh^ca d`Yl Y^Jcf[Ab[YžX]Y^b]WXh^bi f^X]Y^Dc`]nY]ž^gcbXYfb^Ui WX^X]Y^; Ya Y]bgWXUZh^]b$$ $$ \cYa ^A U_YY^JbVYn]YYYb^"8]Y^; fØbXY^ZØf^XUg^JYfgWXk]bXYb^j cbDYfgcbYb^c_ObbYb^j]Y^ZA`h][^gY]b. ^[Ygi bXYY]h`]WXY^DfcV^Ya YžZUa]`]AfY^?cbZ^]_hY^cXYf^gWX^]WXhk Y[^XUg^6YXØfZb]g^bUWXFØW_ni ["'\delta^j]Y^Yb^: A``Ybž^k]Y^Yhk U^VY]^9Xk]b^@'z^YUbXY`hYg^g]WX i a ^A YbgWXYbž^X]Y^Ui Z[fi bX^j cb^9f_fUb_i b[Yb^j cfØVYf[YYYbX cf]Ybh]Yfi b[g`cg^g]bX^cXYf^Ui Z[fi bX^Y]bYf^U_i hYb^?f]gY^<]`ZYVYbOh][Yb"$

a :: U```XYf'Gi WXY'bUWX'9Xk]b'@''k Uf'XYf'9]bgUhn'j cb

Dc`]nY]ghfY]ZYb'i bX'Y]bYa '<i VgWX fUi VYf'bchk YbX][ž'i a 'Y]b

[fÒ» YfYg'5fYU``UVni gi WXYb"'Gc`WXY'A U» bU\a Yb'k YfXYb'\Ài Z][

Yf[f]ZZYbž'k Ybb'XYf'5i ZYbh\U`hgcfh'Y]bYf'j Yfa]gghYb'DYfgcb

i b_`Uf']gh'i bX'XYfYb'G]WXYf\Y]h']b'; YZU\f']gh''8Yf'9]bgUhn'j cb

@i Zhi bhYfghØhni b['_Ubb'XUVY]'\Y`ZYbž'gY`Vgh'gWXk Yf'ni [Àb[`]WXY; YV]YhY'gWXbY``Yf'ni 'YffY]WXYb'i bX'YZZ]n]YbhYf'ni 'Xi fWXgi WXYb''

8]Yg'gd]Y[Y`h'X]Y'Yfbgh\UZhY'<YfUb[Y\Ybgk Y]gY'XYf'Dc`]nY]'k]XYfž

X]Y'ni a 'N]Y``\Uhž'YXY'j Yfa]gghY'DYfgcb'gc'gWXbY```k]Y'a Ò[`]WX

ni 'Z]bXYb"

6Y\ÒfXYba U»bU\a Ybʻi bX ² ZZYbh`]W_Y]hgUfVY]h

8]Y'Ni gUa a YbUfVY]h'nk]gWXYb'XYf'Dc`]nY]'i bX'XYf'² ZZYbh`]WX_Y]h gd]Y`h'Y]bY'YbhgWXY]XYbXY'Fc``Y'VY]'XYf'Gi WXY'bUWX'JYfa]gghYb" \(\phi'j\)]Y`Yb': À``Yb'_Ubb'X]Y'JYfVfY]hi b['jcb'\phiZcfa Uh]cbYb'\ØVYf gcn]U`Y'AYX]Yb'i bX'`c_U`Y'BUWXf]WXhYb'XUni 'VY]hfU[Ybž'kYfhj c``Y <]bk Y]gY'ni 'gUa a Y`b"'\nea ': U```jcb'\9Xk]b'\@''kifXY'X]Y \(^2\) ZZYbh`]WX_Y]hgZU\bXi b[']bhYbg]j '[Ybi hnhž'i a 'XUg'\6Yk i gghgY]b ni 'gWX\AfZYb'i bX'X]Y'\6Yj\O`_Yfi b['i a 'A]h\]`ZY'ni 'V]hhYb"'\9]bY gc`WXY'Jcf[Y\YbgkY]gY'\Uh'g]WX']b'XYf'JYf[Ub[Yb\Y]h'cZh VYk\A\fh"

; Ya À» `XYa `6i bXYg_f]a]bU`Ua h`fb? 5½g]bX`]a `\\\f`&\&\]b

8 Yi hgW\`UbX`]bg[YgUa h`fi bX`\&\"\\$\\$ DYfgcbYb`U`g`j Yfa]ggh

[Ya Y`XYh`k cfXYbž'k cVY]`j]Y`Y`: À``Y`VYfY]hg`]bbYf\U`V`_i fnYf`NY]h

Ui Z[Y_`Àfh`k YfXYb`_cbbhYb"`8]Y`gW\bY``Y`FYU_h]cb`XYf`6Y\ÒfXYbž

_ca V]b]Yfh`a]h`XYf`I bhYfghØhni b[`XYf`6Øf[Yfž`\Uh`]b`j]Y`Yb`: À``Yb

XUni `[YZØ\fhž`XUgg`j Yfa]gghY`DYfgcbYb`gW\bY```i bX`k c\`VY\U`hYb

[YZ bXYb`k i fXYb"`

GhUh]gh]_Yb ni f J Yfa]gghYbgi WX Y]b 8 Yi hgWX `Ub X

8]Y'NU\`Yb'ni a 'H\Ya U'JYfa]gghY']b'8Yi hgW\`UbX'nY][YbžXUgg X]Y'a Y]ghYb'A Y`Xi b[Yb']a '?cbhYl h'j cb'* [YbX`]W\Yb'cXYf À'hYfYb'A YbgW\Yb'ghY\Yb"'GhUh]gh]_Yb'XYg'6?5'VY`Y[YbžXUgg Yhk U'+\$i 'XYf'j Yfa]gghYb'DYfgcbYb']a '5`hYf'j cb'%*'V]g'&%'*J\fYb g]bX"'6Y]'À'hYfYb'JYfa]gghYb']gh'cZh'Y]bY'a YX]n]b]gW\Y'I fgUW\Yž k]Y'8Ya Ybnž'Ui ggW\`U[[YVYbX'ZØf'XUg'JYfgW\k]bXYb"'=a '*J\f &\$&\$'[UV'Yg'`Ui h'6?5'%)"'' %'[Ya Y`XYhY'JYfa]gghYbZÀ``Yž'k cVY] X]Y'@Ògi b['X]YgYf': À``Y']b'-*i 'XYf'G]hi Uh]cbYb'YfZc`[h']gh"'8]Y 9f\O\i b['XYf'5i Za Yf_gUa _Y]h'ZØf'X]Y'H\Ya Uh]_'Xi fW\'Dc`]nY] i bX'A YX]Yb'\Uh'Y]bYb'dcg]h]j Yb'9]bZi gg'Ui Z'X]Y'5i ZZ]bXi b['XYf'jYfa]gghYb'DYfgcbYb"

8]Y``Ub[Y'@]ghY'jcb'jYfa]gghYb'DYfgcbYb'jYfXYih`]W\h'X]Y

Bchk YbX][_Y]h'j cb'dfÀj Ybh]j Yb'A U» bU\a Yb'i bX
5i Z_`Àfi b[gUfVY]h']b'XYf'; YgY``gW\UZh"'5b[Y\Òf][Y'i bX': fYi bXY
gd]Y`Yb'YVYbZU``g'Y]bY'k]W\h][Y'Fc``Yž']bXYa 'g]Y'5bnY]W\Yb'ZØf
Y]bY'a Ò[`]W\Y'; YZÀ\fXi b['ZfØ\nY]h]['Yf_YbbYb'i bX
YbhgdfYW\YbX'\UbXY`b"

8 Y hU]`c

6Ygi WXYb G]Y i bg Ui Z b!U["XY