

NYi [Yb'[Ygi W\h.'; YZÀ\f`]W\Yf VYf\c`j cf[Ub['Ui Z'XYf'@&, %'VY] @Ub[YbXYfbVUW\

5a 'ZfØ\Yb'5VYbX'XYg'&\$"'5i [i gh'&\$&('k i fXY'Ui Z'XYf @UbXghfU» Y'@&, %'nk]gW\Yb'@Ub[YbXYfbVUW\'i bX'K YghYfVi f['Y]b [YZÀ\f`]W\Yf'JYf_Y\fgj cf[Ub['VYcVUW\hYhž'XYf'gck c\``ZØf'X]Y VYhY]`][hYb': U\fYf'U`g'Ui W\'ZØf'UbXYfY'JYf_Y\fghY]`bY\a Yf Ài »Yfgh'VYgcf[b]gYffY[YbX'k Uf"'; Y[Yb'&\$.%)'I\f'VYZi\fYb'XfY] D_k 'X]Y'GhfYW_Yž'U`g'Yg'Xi fW\'Y]b'f]g_UbhYg', VYf\c`a UbÒj Yf'ni Y]bYf'_f]h]gW\Yb'G]hi Uh]cb'_Ua"

8 Ug``YhnhY`XYf`XfY]: U\fnYi [Y'j Yfgi WkhYž'XUg'j cfUi gZU\fYbXY 5i hc`ni 'ØVYf\c`Yb"'K À\fYbX'X]YgYg', VYf\c`j cf[Ub[g'gWkYfhY XUg'j cfXYfghY': U\fnYi ['d`Òhn`]WkYb'bUWk'`]b_g'Ui gž'k Ug'VY]bU\Y ni 'Y]bYa 'Ni gUa a Ybghc» '[YZØ\fh'\ÀhhY"'8Yf': U\fYf'XYg ØVYf\c`YbXYb'5i hcg'fYU[]YfhY'gWkbY``i bX'_cbbhY'Xi fWk'Y]bY; YZU\fYbVfYa gi b['Y]bYb'7fUg\'j Yf\]bXYfb"'BUWk'X]YgYf [YZÀ\f`]WkYb'G]hi Uh]cb'gYhnhY'Yf'gY]bY': U\fh']b'F]Wkhi b[K YghYfVi f['Zcfhž'i bX'X]Y'UbXYfYb': U\fnYi [Y'\]bhYf']\a 'V`]YVYbYVYbZU``q'b]Wkhi bhÀh]["

8 Y f J c f Z U i b X X Y g c Z c f h [Y F Y U h] c b

8]Y`5b[Y`Y[Yb\Y]h`kifXY`ia[Y\YbX`XYf`Dc`]nY]]bgdY_h]cb K YghYfVif[`[YaY`XYh"'8]Y`\]bhYf`XYa 'ØVYf\c`YbXYb': U\fnYi[ZU\fYbXYb`NYi[Yb`k UfYb`hUh_fÀZh][`ibX`\U`ZYb`XYb`6YUahYbž XYb'jYfUbhkcfh`]WXYb': U\fYf`ni 'Z]bXYb"' \DXYa 'g]Y`X]Y`Dc`]nY]`V]g bUWX '? O`V]b[Yb'`chgYbž'ghY``hYb'g]Y'g]WXYfž'XUgg'XYf'VYhfYZZYbXY: U\fYf'gWXbY``'_cbhfc``]Yfh'k YfXYb'_cbbhY"

8]Y'Dc`]nY]'K YghYfVi f['f]W\hYh'Y]bYb'5i Zfi Z'Ub'U``Y'dchYbh]Y``Yb NYi [Yb'XYg'JcfZU``g"'\DyYgcbXYfY'X]Y'DYfgcbž'X]Y'X]Y [YZÀ\f`]W\Y'6fYa gi b['Y]b`Y]hYb'a i gghYž']gh'[YZfU[hž'g]W\'ni a Y`XYb"'K Yf'\DZcfa Uh]cbYb'ni 'X]YgYa 'JYf_Y\fgj cf[Ub['\UVYb_\ObbhYž'k]fX'[YVYhYbž'i bhYf'\\$**''--,\\$)\\$'Ubni fi ZYb"

K Ufi a ']gh'X]YgYf'JcfZU``jcb'6YXYi hib[3

 $\label{thm:linear} $JYf_Y\setminus fgg]W.Yf\setminus Y]h']gh'Y]bY'JYfUbhk cfhi b[\check{z}X]Y'j cb'YXYa :: U\setminus fYfYfbgh'[Ybca a Yb'k YfXYb'gc``hY'''Gc`W.Y'JcfZh``Yž'VY]'XYbYb'X]Y\\ G]W.Yf\setminus Y]h']a 'GhfU"Ybj Yf_Y\f'Xi fW.'fØW_g]W.hg`cgYg'JYf\U`hYb [YZh\fXYh'k]fXž'g]bX'b]W.h'bi f'VYXYb_`]W.ž'gcbXYfb'nY][Yb'Ui W.X]Y'Bchk YbX][_Y]h'i bY]b[YgW.fhb_hYf'5i Za Yf_gUa _Y]h'i bX <math display="block">F @W_g]W.hbU\a Y']a 'GhfU"Ybj Yf_Y\f''8]Y'Dc`]nY]'\Uh'a]h']\fYf gW.bY``Yb'FYU_h]cb'i bX'XYf'9fa]hhii b['XYf'NYi [Yb'Y]bY'k]W.h][YFc``Y'gd]Y`Yb'_ObbYbž'i a 'XUg'DfcV`Ya 'XYf'JYf_Y\fgg]W.Yf\Y]h'ni UXfYgg]YfYb'i bX'dchYbh]Y``Y'F]g]_Yb'ni 'a]b]a]YfYb''$

: U\fYf'gc``hYb']a a Yf'XUfUi Z'UW\hYbž'g]W\Yf'i bX'ØVYf`Y[h'ni ZU\fYb", VYf\c`j cf[Àb[Y'YfZcfXYfb'VYgcbXYfYg'5i [Yba Yf_ž'i a I bZÀ``Y'ni 'j Yfa Y]XYb'i bX'X]Y'G]W\Yf\Y]h'U``Yf'6YhY]`][hYb'ni [Yk À\f`Y]ghYb"'8]YgYf'J cfZU``'Yf]bbYfh'i bg'XUfUbž'XUgg fØW_g]W\hg`cgYg': U\fYb'[YZÀ\f`]W\Y': c`[Yb'\UVYb'_Ubb'i bX'XUgg YXYf'j cb'i bg'Y]bYb'6Y]hfU['ni f'G]W\Yf\Y]h']a 'GhfU»Ybj Yf_Y\f `Y]ghYb'_Ubb"

±bg[YgUa h'nY][h'XYf'JcfZU``'Ui Z'XYf'@k, %ž'k]Y'k]Wkh]['X]Y Ni qUa a YbUfVY]h'nk]gWkYb'XYf'Dc`]nY]'i bX'Ui Za Yf_qUa Yb 6Øf[Yfb']gh"'C\bY'X]Y'gW\bY``Y'FYU_h]cb'XYf'NYi[Yb'\AhhY'XYf JcfZU``'a Ò[`]W\YfkY]gY'gW\kYfk]Y[YbXYfY'?cbgYeiYbnYb [Y\UVh"

$G]WXYfY]hga U bU a Yb']a 'GhfU Ybj Yf_YYf$

8]Y'GhfU*YbjYf_Y\fgg]W\Yf\Y]h']gh'Y]b'nYbhfU`Yg'5b`]Y[Yb'XYf
XYi hgW\Yb'6Y\ÒfXYb"'*À\f`]W\'Yf[fY]ZYb'Dc`]nY]'i bX

JYf_Y\fga]b]ghYf]Yb'j YfgW\]YXYbY'A U*bU\a Ybž'i a 'XUg

I bZU``f]g]_c'ni 'gYb_Yb"'8Uni '[Y\ÒfYb'5i Z_`Àfi b[g_Ua dU[bYbž; YgW\k]bX][_Y]hg_cbhfc``Yb'i bX'X]Y': ÒfXYfi b['j cb

JYf_Y\fgYfn]Y\i b[']b'GW\i 'Yb"'8]Y'JYf_Y\fgi bZJ``ghUh]gh]_'nY][hž
XUgg'X]Y'5bnU\``XYf'JYf_Y\fghchYb']b'8Yi hgW\`UbX']a 'JYf['Y]W\
ni 'XYb'%- +\$Yf'*J\fYb'XYi h']W\'[YgYb_h'k YfXYb'_cbbhY"'K À\fYbX
%- +\$'ØVYf'&%"\$\$\$'DYfgcbYb']b'JYf_Y\fgi bZ\``Yb'ghUfVYbž'k UfYb
Yg']a '*J\f'&\$&&``Ui h'XYa 'GhUh]gh]gW\Yb'6i bXYgUa h'bi f'bcW\
fi bX'&"*\$\$"

9]bY'k]W\h][Y'Fc``Y'gd]Y`Yb'Ui W\'?Ua dU[bYb'ni f'GYbg]V]`]g]Yfi b[XYf'JYf_Y\fghY]`bY\a Yf'ZØf'[YZÀ\f`]W\Yg': U\fj Yf\U`hYbž'k]Y'n"6"U[fYgg]j Yg', VYf\c`Yb'cXYf'5V`Yb_i b['Xi fW\'A cV]`hY`YZcbY"'8]YDc`]nY]'bi hnh'gcn]U`Y'A YX]Ybž'i a 'A b[Y': U\fYf'[Yn]Y`hUbni gdfYW\Yb'i bX'ni 'k UfbYb"

8]YFc``YXYfJYf_Y\fgØVYfkUWXib[

JYf_Y\fgØVYfk UW\i b[']gh'Y]b 'k Y]hYfYf'GW\`ØggY``ni f'9f\Ò\i b[XYf'G]W\Yf\Y]h'Ui Z'XYb 'GhfU» Yb"; YgW\k]bX][_Y]hga Yggi b[Yb i bX'XYf'9]bgUhn'j cb 'F UXUfZU``Yb '\ UVYb 'g]W\ 'U`g'YZZ]n]Ybh Yfk]YgYbž'i a 'XUg': U\fj Yf\U`hYb 'ni 'VYY]bZi ggYb"'@Ui h'Y]bYf Ghi X]Y'XYf'8 Yi hgW\Yb'J Yf_Y\fgk UW\h'\UVYb'fY[Y`a À»][Y; YgW\k]bX][_Y]hg_cbhfc``Yb']b'[YZÀ\fXYhYb'6YfY]W\Yb'ni 'Y]bYa FØW_[Ub['XYf'I bZÀ``Y'i a 'V]g'ni '' \$'DfcnYbh'[YZØ\fh"8]YgY NU\`Yb'j YfXYi h`]W\Ybž'k]Y'k]W\h]['dfÀj Ybh]j Y'A U» bU\a Yb']a GhfU» Ybj Yf_Y\f'g]bXž'i a 'f]g_UbhY': U\fk Y]gY'ni 'j Yfa Y]XYb"

$: U \setminus f_j Y f \setminus U \setminus hY b \cap bX \cdot gY] b Y \cdot 5 i gk] f_i b[Y b \cdot U i Z$

I b ZÀ``Y

: U\fj Yf\U`hYb']gh'Y]bY'XYf'k YgYbh`]W\Yb'I fgUW\Yb'ZØf

J Yf_Y\fgi bZÀ``Y"'5[[fYgg]j Yg': U\fYbž'k]Y'Yg'Ui W\']b'XYa 'J cfZU``
nk]gW\Yb'@Ub[YbXYfbVUW\'i bX'K YghYfVi f['VYcVUW\hYh'k i fXYž

ZØ\fh'\Ài Z]['ni'; YZU\fYb`U[Yb"'@Ui h'Y]bYf'I a ZfU[Y'XYf'5``]Ubn

J Yfg]W\Yfi b[g[fi ddY'[YVYb'' \$'DfcnYbh'XYf'6YZfU[hYb'Ubž'XUgg
g]Y'\Ài Z]['j cb'UbXYfYb': U\fYfb'VYXfÀb[h'k YfXYb"'

Ni gÀhn`]WX`]gh'5V`Yb_i b['Ua 'GhYi Yf'Y]b'k UWXgYbXYg'DfcV`Ya "9]bY'Ghi X]Y'XYg'8Yi hgWXYb'JYf_Y\fgg]WXYf\Y]hgfUhg'nY][hž'XUgg X]Y'Bi hni b['j cb'Ga Ufhd\cbYg'k À\fYbX'XYf': U\fh'Yf\YV`]WX'ni f I bZU``[YZU\f'VY]hfÀ[h"'GWXÀhni b[Yb'ni Zc`[Y'g]bX'Yhk U'&\$'DfcnYbh XYf'JYf_Y\fgi bZÀ``Y'Ui Z'5V`Yb_i b['ni fØW_ni ZØ\fYbž'k Ug'X]Y 8f]b[`]WX_Y]h'j cb'A U» bU\a Yb'ni f'GYbg]V]`]g]Yfi b['i bhYfghfY]WXh"

² ZZYbh`]WX Yg 6Yk i gghgY]b i bX 6Yf]WX hYfghUhhi b[

8]Y a YX]U`Y '6Yf]W\ hYfghUhhi b['ØVYf'J Yf_Y\fgi bZ\``Y 'hf\\[A] h YVYbZU``g'ni f'GYbg]V]`]g]Yfi b['XYf'^2 ZZYbh`]W_Y]h'VY]"'K Ybb'gc`W\Y J cfZ\[A]``Y']b'XYb'BUW\f]W\ hYb'VY\UbXY`h'k YfXYb\[Z\[A]\Yg'\cZh'ni Y]bYa '5bgh]Y['XYf'8]g_i gg]cb'\[ØVYf'J Yf_Y\fgg]W\Yf\Y]h'i bX'X]Y B chk YbX][_Y]h'j cb'J Yf\U`hYbg\[AbXYfi b[Yb"'? Ua dU[bYb]bYf\U`V'XYf'A YX]Yb\[Z'\X]Y'Ui Z'j YfUbhk cfhi b[gVYk i gghYg': U\fYb\]bk Y]gYb\[Z'\G]bX'\[Z\[A]\Y': \OfXYfi b['Y]bYf'\G]\W\YfYfYb JYf_Y\fg_i `hi f'i bYf'\[A]\G]\W\"