

# $Hf\varnothing ZZY`!GYbgUh]cb. < i b X @Yc YbhXYW_h gY`hYbYb ? i d ZYffchYb GWX`Y]a hf<math>\varnothing ZZY`$

HfØZZY`gi W\\i bX`@Yc`YbhXYW\_h`gY`hYbYb`?idZYffchYbGW\`Y]a hfØZZY``]a '6UmYf]gW\Yb`KU`X` 'YfghYf`BUW\kY]g`XYf5fh`]b`XYf`FY[]cb"

 $= a -6 \ UmY f]gWX Yb'K U'X' \setminus Uh'XYf'Hf \emptyset ZZY `gi WX \setminus i b X' @Yc'Y]bYb \\ VYXYi hYbXYb': i bX'[Ya UWX hž'XYf'b]WX h'bi f'ZØf'X]Y'FY[]cbž \\ gcbXYfb'Ui WX'ZØf'X]Y'D]`nZcfgWXi b['j cb'[fc<math>>$ Yf'K]WX h][\_Y]h']gh'' \\ 8 Yf' < i bXž'XYf'XYa 'A m\_c`c[Yb'DYhYf'? UfUgWX'Ub[Y\Ofhž YfgWX b ØZZY`hY'Y]bYb'?i dZYffchYb'GWX`Y]a hf ØZZY`fA Y`Ubc[UghYfhi VYf]Zcfa ]gŁž'XYf'Yfgha U`g']b'X]YgYf'; Y[YbX'bUWX[Yk]YgYb k i fXY''

@YcbUfXi gž'Ui W\ '@Yc'[YbUbbhž'k i fXY'ZØf'gY]bY
Ui » Yf[Yk Ò\b`]W\Yb': À\][\_Y]hYb'ni f'HfØZZY`gi W\Y']b'XYb'`YhmhYb
\(\famble \text{Y}\text{b}'\text{a}\text{b}'\text{Y}\text{b}'\tex

### 9fZcfgWXib['XYf'HfØZZY`.'9]b'k Yfhj c``Yf 6Y]hfU['nif'A m\_c`c[]Y

8]Y'[YZ bXYbYb': fi Wkh\_OfdYf'k UfYb'Yhk U'nk Y]'NYbh]a YhYf'[fc» i bX'\UhhYb'Y]bYb']bhYbg]j Yb'; Yfi Wkž'XYf'Ub'j YffchhYbXY
Nk]YVY'b'Yf]bbYfhY''8]Yg'a UWkh'g]Y'ZØf'[i h'Ui g[YV]'XYhY'<i bXY
j Yf[ `Y]Wkgk Y]gY'`Y]Wkh'Ui ZZ]bXVUf''@Ui h'Y]bYa 'GdfYWkYf'XYf
BUh]cbU'dUf\_j Yfk U`hi b[ 'j YfV]f[h'g]Wk'\]bhYf'XYffY]bn][Ufh][Yb
: À\][ \_Y]h'j cb'@Yc'j ]Y`'HfU]b]b['i bX'9fZU\fi b[''

#### GYbgUh]cbY``Yf'BUWkY]g'ZØf'X]Y'FY[]cb

8]Y'6YXYi hi b['X]YgYg': i bXYg'k]fX'Xi fW\'Y]bY'a]\_fcg\_cd]gW\Y I bhYfgi W\i b['i bhYfghf]W\Ybž'X]Y'a]hhYfk Y]'Y'VYgh\h][h\\Uh\ž'XUgg Yg'g]W\'hUhg\W\`]W\'i a 'XYb'?i dZYffchYb'GW\`Y]a hf\ØZZY`\\UbXY\h' 8]YgY'9bhXYW\_i b['\Uh'['Y]W\'a Y\fYfY'8]a Ybg]cbYb.'9g']gh'b]W\h bi f'XYf'YfghY'BUW\kY]g'X]YgYf'Hf\ØZZY`Ufh']a 'BUh]cbU`dUf\_\z\ gcbXYfb'Ui W\']b'XYf'[YgUa hYb'FY[]cb'6\O\a Yfk U`X''', VYfX]Yg']gh Yg'XYf'nk Y]hY'BUW\kY]g'X]YgYf'5fh']b'[Ubn'6UmYfb''8]Y 9bhXYW\_i b['hf\[a\][h'gca]h'XUni 'VY]\z\X]Y'6]cX]j Yfg]h\[a\hXYf'FY[]cb i bX'XUg'K]ggYb'\@VYf']bZc`[Y'XYf'\_\]a Uh]gW\Yb'6YX]b[i b[Yb a \O[\`]W\Yfk Y]gY'j Yf\[a\bXYfhY'D]`ndcdi `Uh]cbYb'ni 'Yfk Y]hYfb''

8 Ugʻ5i ZgdØfYbʻj cbʻHfØZZY`bʻghY``hʻY]bYʻgdUbbYbXY < YfUi gZcfXYfi b[ˈXUfži bXˈX]Yʻ5fVY]hʻj cbʻ< i bXYbʻk ]Yʻ@Ycʻ]gh XYg\U`Vʻi bj Yfn]W\hVUf"ʻ\bʻY]bYfʻŸfUžʻ]bʻXYfʻX]Yʻ5i gk ]f\_i b[YbʻXYg ?`]a Uk UbXY`gʻUi ZʻX]YʻB Uhi fʻ]a a Yfʻa Y\fʻ]bʻXYbʻ: c\_i gʻfØW\_Ybžʻ]gh Ygʻk ]W\h][žʻgc`W\Y'Y]bn][Ufh][Ybʻ9bhXYW\_i b[Ybʻni 'ZY]Yfbʻi bX XYfYbʻ6YXYi hi b[`\Yfj cfni \YVYb"

8]Y'9IdYfhYb'\cZZYbž'XUgg'@]cbg'Ui »Yf[Yk Ò\b`]W\Yf': ibX'b]W\hbif'XUg'\bhYfYggY'Ub'XYf'HfØZZY`gi W\Y'k YW\_hž'gcbXYfb'Ui W\'XUg 6Yk igghgY]b'ZØf'XYb'9f\U`h'jcb'2\_cgmghYa Yb'ghÀf\_h"'8Yf 6UmYf]gW\Y'K U`Xž'VY\_Ubbh'ZØf'gY]bY'VYY]bXfi W\_YbXY'BUhif'ibXfY]W\Y'5fhYbj]Y`ZU`hž'\_ÒbbhY'\_ØbZh]['Y]b'NYbhfia'ZØf'X]Y HfØZZY`ZcfgW\ib['k YfXYb"

@Yc']gh'gca ]h'b]W\h'bi f'Y]b'Y]bZUW\Yf'<i bXž'gcbXYfb'Y]b'YW\hYfGW\Uhngi W\Yf ž'XYfž'bYVYb'gY]bYa'6Yg]hnYf'DYhYf'?UfUgW\ž

Y]bYb'k Yfhj c``Yb'6Y]hfU['ni f'k]ggYbgW\UZh`]W\Yb'9bhXYW\_i b[
`Y]ghYh"8]Y'?ca V]bUh]cb'Ui g'Y][Ybg'Ui g[YV]`XYhYb'<i bXYb'i bX
`Y]XYbgW\UZh`]W\Yb': cfgW\Yfb'\_ObbhY'GW\`ØggY``ni 'bcW\`kY]hYfYbž
V]g`Ub['i bVY\_UbbhYb'GW\ÀhnYb'XYf'BUhi f'k YfXYb"

### 9]b'<i bX'U`g'9bhXYW\_Yf.'8]Y ni\_i bZhgk Y]gYbXY'Fc``Y'XYf HfØZZY`gi WX\i bXY

8]Y`]bbcj Uh]j Y`<YfUb[Y\Ybgk Y]gYž'HfØZZY`gi W\\i bXY`]b`X]Y
: cfgW\i b[ 'Y]bni VYn]Y\Ybž'\_ObbhY`Y]bY`J cfV]`X\overline{Z}j b\_h]cb`\overline{Z}jf
UbXYfY`FY[ ]cbYb`\@VYfbY\a Yb"`8]Y`BUhi f`V]YhYh`gW\]Yf
i bYbX`]W\Y`A O[`]W\\_Y]hYb`ni f`9bhXYW\_i b[ 'bYi Yf`5fhYb`i bX`ni f
J YfVYggYfi b[ 'i bgYfYg`J YfghAbXb]ggYg`XYf`2 \_cgmghYa Y"'@Yc`\Uh
a ]h`gY]bYa``VYgcbXYfYb`: i bX\overline{Z}XYf`]a``6UmYf]gW\Yb`K`U`X`j cb
[ fc\(^{Y}Yf`6YXYi\) hi b[ ']gh\overline{Z}Y]Y`HØf`ni 'bYi Yb``9f\_Ybbhb]ggYb
Ui Z[ Yghc\(^{Y}Yb\overline{Z}Yi\) bX'bYi Y`DYfgdY\_h]j Yb`Ui ZhY][ Yb``\_ObbYb"

8 Yf?i dZYffchY 'GWX `Y]a hfØZZY `ž k ]ggYbgWX UZh`]WX 'U`g'A Y`Ubc[ UghYfhi VYf]Zcfa ]g'VY\_Ubbhž']gh'Y]b 'gY`hYb'j cf\_ca a YbXYf'D]`nž'XYf'Y]bYVYgcbXYfY 'F c``Y']a '2 \_cgmghYa 'gd]Y`h" 8 ]YgY '5 fh'[ Y\Òfh'ni f; fi ddY 'XYf'i bhYf]fX]gWX Yb'D]`nYž'X]Y 'cZh'gma V]ch]gWX Y6Yn]Y\i b[ Yb'a ]h'DZUbnYb'Y]b[ Y\Yb"'HfØZZY``g]bX'ZØf']\fY'Fc``Y']bXYf'A m\_cff\]nU'VY\_Ubbhž'Y]bYf'JYfV]bXi b[ 'nk ]gWXYb'D]`nYb'i bXDZUbnYbk i fnY`bž'X]Y'XUg'K UWXghi a 'i bX'X]Y'BÀ\fghcZZUi ZbU\a Yjcb'6Ài a Yb'j YfVYggYfh" 8 Ug'5i ZgdØfYb'X]YgYf'HfØZZY``YfZcfXYfhgdYn]Y``Y': À\][ \_Y]hYbž'gck c\``j cb'XYf'BUhi f'U`g'Ui WX'Xi fWX[YgWXi `hY'H]YfY'k ]Y'@Ycž'XYb'Gi WX\i bX"

#### K ]fhgWX UZh`]WX Y 6 Y X Y i hi b [ 'j c b 'HfØZZY `b

8]Y '9bhXYW\_i b[ 'gY`hYbYf'HfØZZY`UfhYb'\_Ubb'Ui W\ 'k ]fhgW\UZh`]W\Y 5i gk ]f\_i b[ Yb'\UVYb"'HfØZZY`'g]bX'Y]bY'8Y`]\_UhYggY'i bX'\_ÒbbYb\c\Y'A Uf\_hdfY]gY'Yfn]Y`Yb"'8Yf'K Yfh'j cb'HfØZZY`b'\Àb[ h'j cbj YfgW\]YXYbYb': U\_hcfYb'UVž'k]Y'XYf'5fhž'XYf'Ei U`]hÀh'i bX'XYf JYfZØ[ VUf\_Y]h"'6Y]'Y]bYa 'gdYn]Z]gW\Yb': i bX'k]Y'XYa '?i dZYffchYb

GWX`Y]a hfØZZY``ghY``h`g]WX`X]Y': fU[YžcV`X]YgYf`D]`n`Ui WX`]b`XYf; Ughfcbca ]Y`JYfkYbXi b[`Z]bXYb`k]fX`cXYf`cV`Yf`Y\Yf`ZØf`X]Yk]ggYbgWXUZh`]WXY': cfgWXi b[`j cb`±bhYfYggY`]gh"

±0.8 Yi hgWX `Ub X '[ Yk ]bbh'X]Y 'HfØZZY`ni WX h'ni bY\a Yb X 'Ub
Dcdi `Uf]hÀhž'i bX 'Ui WX 'XYf'6 UmYf]gWX Y 'K U`X '\_ObbhY 'g]WX 'U`g
dchYbn]Y ``Yg'5bVUi [ YV]Yh 'ZØf'\_ca a Yfn]Y ``Yb 'HfØZZY`UbVUi
Ybhk ]W\_Y`b" '±0]h]Uh]j Yb 'ni f'NØWX hi b[ 'i bX '9f\U`hi b[ 'j cb 'HfØZZY`b
\_ObbhYb 'XUVY]'\Y`ZYbž'gck c\``X]Y '6]cX]j Yfg]hÀh'ni 'ZOfXYfb'U`g
Ui WX 'k ]fhgWX UZh`]WX Y '7\Ub WYb'ZØf'X]Y 'FY[ ]cb'ni 'gWX UZZYb"
A Y`Xi b[ Yb'ØVYf'ghY][ YbXYg'±bhYfYggY'Ub'XYf'D]`n\_i `hi f']b
8 Yi hgWX`UbX'`UggYb'XUfUi Z'gWX`]Y » Ybž'XUgg'@UbXk ]fhY'i bX
A ]\_fc`c[ Yb'ni bY\a YbX'ni gUa a YbUfVY]hYbž'i a 'X]Y
HfØZZY`]bXi ghf]Y'ni 'VY`YVYb"'

## : cfgWXib['ibX'A m\_c`c[]Y']a '6UmYf]gWXYb K U`X

8 UfØVYf`\]bUi g`g]bX`gc`W\Y`9bhXYW\_i b[Yb`k]W\h][`ZØf`X]Y GYbg]V]`]g]Yfi b[`XYf`² ZZYbh`]W\\_Y]h`ZØf`X]Y`6YXYi hi b[`j cb`D]`nYb]a '² \_cgmghYa "'Ÿ\b`]W\`k]Y`VY]`XYf`9bhXYW\_i b[`UbXYfYf`gY`hYbYbGdYn]Yg`ZØ\fh`XUgʻ5i ZZ]bXYb`gc`W\Yf`9l Ya d`UfY`cZh`ni 'Y]bYa[Ygh]Y[YbYb`+bhYfYggY`Ub`XYf`l a k Y`hl`i bX`BUhi fgW\i hnUfVY]h"8]Y`9]bVYn]Y\i b[`j cb`<i bXYb`]b`X]Y`: cfgW\i b[`YfÒZZbYh`ni XYabYi Y`DYfgdY\_h]j Ybž`XU`X]YgY`H]YfY`VYgcbXYfg`[i h`XUf]b`g]bXžgi Vh]`Y`l bhYfgW\]YXY`]b`; YfØW\Yb`ni 'Yf\_YbbYbž`X]Y`A YbgW\Yb a Ò[`]W\Yfk Y]gY`Ybh[Y\Yb`k ØfXYb"

## 6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'b!U[ "XY