

?fmdhck À\fi b[Yb]a 5i Zk]bX.K UgX]Y Ni _i bZhZØf5b`Y[YfVYfY]h\A`h

?fmdhck À\fi b[Yb'Yf`YVYb'Y]bY'VYa Yf_Ybgk YfhY'9f\c`i b[XUb_'bYi Yf'9H: g"'9fZU\fYb'G]Yž'k Y`WXY'5`hWc]bg'dfcZ]h]YfYb__ObbhYb"

8]Y'K Y'h'XYf'?fmdhck À\fi b[Yb''YVh'Ui Z'i bX'n]Y\h'k]YXYf'X]Y
5i Za Yf_gUa _Y]h'j cb' \(\phi\) YghcfYb'Ui Z'g]W\"'BUW\'Y]bYf'D\UgY'XYf
I bg]W\Yf\Y]h']b'XYf'['cVU'Yb':]bUbn'UbXgW\UZh'\UVYb'X][]hU'Y
J Yfa Ò[Ybgk YfhY'k]Y'6]hWc]b'i bX'9h\YfYi a 'Y]bY'VYY]bXfi W_YbXY
9f\c'i b['[YnY][h'' \(\phi\) XYb'YfghYb'A cbUhYb'XYg'\(\phi\)\\fYg'\&\\$\&('\UVYb'\)g]W\'X]YgY'K À\fi b[Yb'i bhYf'XYb'VYghYb'5b`U[Yb'YhUV']Yfhz'i bX
Ui W\'X]Y'5_h]Yb_i fgY'j cb':]fa Yb'k]Y'Bj]X]U'hfU[Yb'ni 'X]YgYa
HfYbX'VY]"

K Ugʻg]Wk '\]YfʻUVgd]Y`hžʻ\UhʻXUgʻDchYbn]Uʻžʻk Y]hfY]WkYbXY

J YfÀbXYfi b[YbʻZØfʻXYbʻA Uf_hii bXʻX]Yʻ5b`Y[Yfʻni 'Vf]b[Yb"'8]Yg

]ghʻ]bgVYgcbXYfYʻj cfʻXYa '<]bhYf[fi bX`XYfʻØb[ghYb

fY[i`Uhcf]gWkYbʻŸbXYfi b[Ybʻni 'j YfghY\YbžʻX]YʻXUni 'VY][YhfU[Yb
\UVYbž'XUggʻ?fmdhck À\fi b[Ybʻa Y\f′; `Ui Vk ØfX][_Y]h'i bX

G]WkYf\Y]hʻ]bʻXYbʻ5i [YbʻXYfʻ≠bj YghcfYbʻ[Yk]bbYb"

$8 Y 9bhk W_i b[i cb 6]bUbW 7c]b f6B6Ł$

9]b VYgcbXYfg Ui ZZÀ``][Yg 6Y]gd]Y `ZØf XYb 5i ZgWk k i b[']a ?fmd hck À\fi b[ga Uf_h]gh 6]b Ub W '7c]b 'f6B 6Ł" I fgd fØb[`]W '&\$%+U`g I h]`]hm! Hc_Yb 'ni f '6YnU\`i b['j cb '< Ub XY`g[YVØ\fYb 'Ui Z'XYf VY`]YV hYb '?fmd hck À\fi b[gV ÒfgY '6]b Ub W 'Y]b[YZØ\fhž \ Uh '6B 6 a]hh`Yfk Y]`Y 'Ub '6YXYi hi b['[Yk cbbYb" '9g 'k]fX ']b 'nU\`fY]W Yb 5bk Yb Xi b[Yb '[Ybi hnhž j cb 'NU\`i b[Yb 'ØVYf '6i W i b[Yb 'V]g \]b ni '\Dj Ygh]h]cbYb" 'GY]b 'K Yfh \ Uh 'g]W 'i a 'VYa Yf_Ybgk YfhY', - ž* 'i

Yf\O\hžk Ug`XUg`JYfhfUi Yb`XYf`Bi hnYf`]b`X]YgY`?fmdhck À\fi b[k]XYfgd]Y[Y`h"

8]Y 'HUhgUW Yž XUgg 'g]W '6B6']b 'gc 'j]Y 'Y b '6Y fY]W Y b 'Y hUV `]Y fh'\Uhž nY][hž XUgg ']a '? fmd hck À\fi b[ggY_hcf'b]W h'bi f'6]hWc]b 'i bX 9h\YfYi a 'j cb'6YXYi hi b['g]bX"'8]YgY'8]j Yfg]Z]n]Y fi b['_ÒbbhY'XUg \diphYfYggY'Ub'UbXYfYb'5`hWc]bg'k YW_Yb'i bX'X]Y'8 mbUa]_']b'XY f [YgUa hYb'6fUbW Y'ghÀf_Yb"

$6'W_U U ZXYYnk YhY'>U FYg À'ZhY$

 $8]Y A Uf_hYbhk]W_i b[Yb ZØ hYb ni XYa : Un]hž XUgg 5b Y[Yf]b XYf nk Y]hYb &U hYg \A ZhY && (a]hV]g XUhc i bYfk UfhYhYb 9bhk]W_i b[Yb fYW bYb _ObbYbž]bg VYgcb XYfY]b 6Yni [Ui Z 5 hWc]bg 8 i fW X]Y 9]b ZØ hi b[i cb Gdch! 9h \ Yf! 9H: g k]f X Y]bY bYi Y Y fU Y]b[YY]hYhž X]Y a O[`]W Yfk Y]gY Ui W Ub XYfY X][]hU Y K A hi b[Yb]b XYb J cf XYf[fi b X fØ W_Yb _ObbhY " 9 I d YfhYb k]Y X]Y j Yfg]YfhY 6 c W_W U]b! J Yf ZYW hYf]b ? Uj]hU; i d hU gY \ Yb 5 bnY]W Yb XUZØ f z XUgg XYf b A W ghY 5 hWc]b! 6 cca VYj c fghY h"$

5`hYfbUh]j Yb`k]Y`Gc`UbUž`F]dd`Y`i bX`@]hYWc]b`ghY\Yb`Ui Z`XYf`@]ghYa Ò[`]W\Yf`?UbX]XUhYbž`X]Y`j cb`X]YgYf`Yf\Ò\hYb`5i Za Yf_gUa _Y]hdfcZ]h]YfYb`_ÒbbhYb"`8]YgY`K À\fi b[Yb`_ÒbbhYb`ni gUa a Yb`a]hYhUV`]YfhYb; fÒ»Yb`Y]bY`[fÒ»YfY`\$j YghcfYbVUg]g`Ubn]Y\YbžkcXi fW\XUg`[YgUa hY`?fmdhc!²_cgmghYa `dfcZ]h]YfYb`k ØfXY"

 $5 ``YfX]b[g'[]Vh'Yg'Ui WX'<YfUi gZcfXYfi b[Yb'''FcVYfh'A]hWXb]W_j cb'6`UW_FcW_'k UfbhY'XUj cfž'XUgg'Y]bY'Yf\O\hY'5bnU\``Ub 6]hWc]b!9H: g'b]WXh']b'[fY]ZVUfY'BA\Y'fØW_h'''8]Y'I bg]WXYf\Y]h]bbYf\U`V'XYf'fY[i `Uhcf]gWXYb'FU\aYbVYX]b[i b[Yb'_ObbhY dchYbh]Y``Yb'=bj YghcfYb']a 'K Y['ghY\Ybž'k Ug'X]Y A Uf_hVYX]b[i b[Yb'VYY]bZ'i ggh''$

Ni XYa '_ObbhY'XYf'Ni [Ub['ni 'X]YgYb'bYi Yb'\Dghfi a YbhYb'k]Y 6]hWc]b!9H: g'Xi fW\hfUX]h]cbY``Y'6Ub_Yb'k Y]hYf'[YZOfXYfh'k YfXYb" 9g'[]Vh'VYfY]hg'6Yf]W\hYž'XUgg'[fc»Y'6Ub_Yb'k]Y'A cf[Ub'GhUb`Ym VYfY]h'g]bXž']\fY'6YfUhYf'XYb'JYf_Ui Zj cb'6]hWc]b!9H: g'ni [YghUhhYb"'8]Yg'_ObbhY'XYb'Ni [Ub['ni '?fmdhck À\fi b[Yb'ZØf'X]Y

 $VfY]hY^2 ZZYbh^3]WX_Y]h^YfbghVUZh^Yf^Y]WXhYfb^ibX^X]Y^U^*[YaY]bY 5_nYdhUbn^ijcb^X][]hU^Yb^JYfa O[YbgkYfhYb^kY]hYf^YfVOVYb''$

8]Y: cfhgWkf]hhY:Ui Z:X]YgYa ; YV]Yhig]bX:b]Wkhibi fiZØfi±bj YghcfYb j cb:6YXYi hi b[žigcbXYfb:_ObbhYb:Ui Wk:X]Y:U``[Ya Y]bY ±bj Ygh]h]cbg_i `hi fiVYY]bZi ggYb":=a a Yfia Y\fiA YbgWkYb:Yf_YbbYb XUg:DchYbn]U``j cb:X][]hU`Yb'K À\fi b[Yb'i bX:XYfYb'Fc``Y']a ni _ØbZh][Yb::]bUbngmghYa "

FY[i`Uhcf]gWXY'l bhYfghØhni b['U`g K YbXYdi b_h

 $\label{thm:continuous} $$J \cb^{(fc)} Yf'K \]Wh][\] Yf'Y \] Yf'Y \fruin Yf$

8]YgY'BYi][_Y]hYb'\UVYb'XUg'DchYbh]U`ž'X]Y'8mbUa]_']a ?fmdhck À\fi b[ggY_hcf'ni 'j YfÀbXYfb"'8]Y'HUhgUWXYž'XUgg'X]Y Z]bUbn]Y``Yb'5i Zg]WXhgVY\OfXYb'VY[]bbYbž'X]Y'F]WXhi b[j cfni [YVYbž'_ObbhY'UbXYfYb'@ÀbXYfb'U`g'A cXY```X]YbYb'i bX'X]Y [`cVU`Y'5_nYdhUbn'j cb'?fmdhck À\fi b[Yb'ZOfXYfb"

8 Y f 'XmbUa]gWk Y '? fmd hck À\fi b[ga Uf_h 'n Y][hž X Ugg' ≠ j Yghc f Y b V Y f Y]h 'g]b X ž 'g]Wk 'Ui Z'b Y i Y 'H f Y b X g 'Y]b ni `Ugg Y b ž i b X 'X Ugg 'X][]hU`Y J Y fa Ò[Y b gk Y f h Y k Y]h Y f \]b 'Y]b Y k]Wk h][Y 'F c ``Y ']b 'X Y f :]b Ub nk Y`h gd]Y`Y b '_Òb b h Y b "

$A\ Uf_h[\ YZ\emptyset\backslash `Y'i\ bX'X]Y'Fc``Y'j\ cb'=bj\ YghcfYb$

 \pm 'U'''XYa '[]Vh'Yg'YXcW\'Ui W\'g_Ydh]gW\Y'Gh]a a Yb"'Hfchn'XYf dcg]h]j Yb'9bhk]W_`i b[Yb'k UfbYb'Y]b][Y'9l dYfhYbž'XUgg'Yg

a Ò[`]WXYfk Y]gY'b]WXh'j]Y`Y'bYi Y'?fmdhck À\fi b[g!9H:g'[YVYbk]fX"'8]Yg'_ÒbbhY'X]Y'A Uf_h[YZØ\`Y'VYY]bZi ggYb'i bX'X]Y

♣bj Ygh]h]cbgVYfY]hgWXUZh'XYf'5b`Y[Yf'XÀa dZYb"'9g'k]fX'k]WXh][
gY]bž'X]Y'_ca a YbXYb'A cbUhY'[YbUi 'ni 'VYcVUWXhYbž'i a 'ni
gY\Ybž'cV'g]WX'X]YgYf'HfYbX'ZcfhgYhnh'cXYf'cV'UbXYfY': U_hcfYb
9]bZi gg'XUfUi Z'bY\a Yb"

5`hWc]b!FU``mY']a '?ca a Yb

±g[YgUa h'XYi hYh'U``Yg'XUfUi Z'\]bž'XUgg'X]Y'_ca a YbXYb'A cbUhY ZØf'XYb'?fmdhck À\fi b[ga Uf_h'YbhgW\Y]XYbX'gY]b'k YfXYb"'8Ug DchYbn]U``ZØf'Y]bY'5`hWc]b!FU``mY']gh'[Y[YVYbž'i bX'XYf'A Uf_h'k]fX [YbUi 'VYcVUW\hYbž'k Y`W\Y'K À\fi b[Yb'U`g'bÀW\ghY']bg FUa dYb`]W\h'hfYhYb'_ÒbbhYb"'8]Y'j]Y`ZÀ`h][Yb'9bhk]W_`i b[Ybž'X]Y g]W\'UVnY]W\bYbž'a UW\Yb'[YgdUbbh'Ui Z'X]Y'bÀW\ghYb'GW\f]hhY'XYf 6fUbW\Y'i bX'X]Y'5bdUggi b[Ybž'X]Y'Ui Z'X]Y'5b`Y[Yf'ni _ca a Yb_ObbhYb"

?fmdhck À\fi b[Yb'i bX']\fY'Fc``Y']b'XYf [`cVU`Yb'K]fhgWXUZh

8]YgY'JYfÀbXYfi b[Yb'\UVYb'Ui W\'dc`]h]gW\Y'=a d`]_Uh]cbYb'''=b j]Y`Yb'@ÀbXYfb'gY\Yb'FY[]Yfi b[Yb']b'XYf'9]bZØ\fi b['j cb

? fmd hc k À\fi b [Yb 'gc k c\`'7\Ub\\Yb 'U`g'Ui \\X\'F]g]_Yb ž' k Ug'ni i bhYfg\\X\]YX`]\\X\Yb '5bg\AhnYb']b'\XYf'FY[i`]Yfi b ['i b\X'\DhY[fUh]cb \]b'\X]Y' k]fhg\\X\UZh`]\\X\Y'\D\ZfUghfi _hi f'\Z\D\fh''

HYW bc`c[]gW Y'9bhk]W_`i b[Yb'i bX =bbcj Uh]cbYb

8]Y'hYW bc`c[]gW Yb': cfhgW f]hhY']a '6YfY]W 'XYf'6`cW_W U]b'i bX XYnYbhfU`Yf':]bUbn]Yfi b['fBY:]Ł'g]bX'Y]b'k YgYbh`]W Yf': U_hcf \]bhYf'XYa 'K UW ghi a 'XYf'?fmdhck À\fi b[Yb" \\
\[\bcj Uh]cbgdfc Y_hY'k]Y'9h\YfYi a '&"\$'i bX' \\
\[@UmYf!&!G_U`]Yfi b[g`Ògi b[Yb'n]Y`Yb'XUfUi Z'UVŽ'X]Y'9ZZ]n]Ybn i bX'G_U`]YfVUf_Y]h'XYf'BYhnk Yf_Y'ni 'j YfVYggYfb"'8]YgY \\
9bhk]W_`i b[Yb'\UVYb'b]W h'bi f'XUg'DchYbn]U`Ž \\
HfUbgU_h]cbg_cghYb'ni 'gYb_Ybž'gcbXYfb'YfÒZZbYb'Ui W 'bYi Y \\
5bk YbXi b[ga Ò[`]W _Y]hYbž']bgVYgcbXYfY']a '6YfY]W 'XYf \\
]bhY``][YbhYb'JYfhfÀ[Y'i bX'XYnYbhfU`Yb'5bk YbXi b[Yb'fK5ddgŁ"

I a `X]Y`J cfhY]`Y`XYf`6`cW_W\U]b!HYW\bc`c[]Y`k Y]hYf`ni
a UI]a]YfYbž']bj Ygh]YfYb`I bhYfbY\a Yb`i bX`FY[]Yfi b[Yb
ni bY\a YbX`]b`: cfgW\i b[g!`i bX`9bhk]W_`i b[gdfc`Y_hY"`9]b
6Y]gd]Y``\]YfZØf`]gh`XUg`?cbgcfh]i a `8@H`@UV ž`XUg`]bbcj Uh]j Y
8]YbghY`a]h`8]ghf]Vi hYX!@YX[Yf!HYW\bc`c[]Y`ZÔfXYfh`i bX`gca]h
hfUX]h]cbY``Y`; YgW\ÀZhga cXY``Y`X]gfi dh]YfYb`_ÒbbhY"

A Uf_hUbU`mqY'i bX'HfYbXq

**/\f	A Uf_h_Ud]hU`]g]Yfi b[<uidhu_hyify< th=""></uidhu_hyify<>
	f]b'6]``]cbYb'l G!	
	8 c``llfł	

&\$&\$	\$ ž&	6]hWc]bž'9h\YfYi a
&\$&%	&ž)	6]hWc]bž9h\YfYiaž
		6]bUbWY [.] 7c]b
&\$&'	&ž) Ž	6]hWc]bž9h\YfYiaž
		Gc`UbU

8]Y'AUf_hUbU`mgYb'nY][Ybž'XUgg'X]Y'AUf_h_Ud]hU`]g]Yfib['jcb?fmdhckÀ\fib[Yb']b'XYb'`YhnhYb'\\fYb'ghUf_'ni[YbcaaYb'\Uh"

5_hi Y``Y`GW\Àhni b[Yb`[Y\Yb`XUj cb`Ui gž`XUgg`X]Y

A Uf_h_Ud]hU`]g]Yfi b[`V]g`9bXY`&\$&(`k Y]hYf`ghY][Yb`_ObbhYž
]bgVYgcbXYfY`Xi fW\`XUg`ghY][YbXY`=bhYfYggY`Ub`bYi Yb

HYW\bc`c[]Yb`i bX`5bk YbXi b[Yb`]a `?fmdhck À\fi b[gO_cqmghYa "

6YgcbXYfg'VYa Yf_Ybgk Yfh']gh'X]Y'ni bY\a YbXY'K Y]ggU[i b[ž XUgg'XUg' \(\alpha\) hYfYggY'Ub'i a k Y`hZfYi bX`]W\YfYb'6`cW_W\U]b!

HYW\\\ bc`c[]Yb'i bX'bUW\\U`h][Yb'? fmdhck \(\alpha\) fi b[gdfc\Y_hYb']b'XYb

_ca a YbXYb'\(\alpha\) fYb'ghY][Yb'k]fX"'I a ZfU[Yb'nY][Ybž'XUgg'j]Y`Y

\(\alpha\) YghcfYb'ni bY\a YbX'K Yfh'Ui Z'BUW\\U`h][_Y]h'`Y[Ybž'k Ug'ni
Y]bYa '5bgh]Y['YbhgdfYW\YbXYf'Dfc\Y_hY'ZØ\fYb'_\ObbhY''

Gcn]U`Y'5i gk]f_i b[Yb XYf'?fmdhc!5Xcdh]cb

8]Y'5_nYdhUbn'j cb'?fmdhck À\fi b[Yb'\Uh'b]W\h'bi f
k]fhgW\UZh]W\Y': c'[YbžgcbXYfb'_Ubb'Ui W\h]YZ[fY]ZYbXY'gcn]U'Y
5i gk]f_i b[Yb'\UVYb"'=\bgVYgcbXYfY'XYf'Ni [Ub['ni
:]bUbnX]Ybgh'Y]ghi b[Yb'ZØf'i bVUb_]YfhY'6Yj Ò'_Yfi b[g[fi ddYb_ObbhY'g]W\jYfVYggYfb"'@Ui h'Y]bYf'Ghi X]Y'XYf'K Y'hVUb_'\UVYbZUgh'%\z+A]'\]UfXYb'A YbgW\Yb'k Y'hk Y]h'_Y]bYb'Ni [Ub['ni
fY[i 'ÀfYb'6Ub_X]Ybgh'Y]ghi b[Yb"'?fmdhck À\fi b[Yb'_ObbhYbX]YgY'6Uff]YfYb'UVVUi Yb'i bX'XYb'A YbgW\Yb'X]Y'A Ò[\]W_Y]h
[YVYb\z*Ub'XYf'['cVU'Yb'K]fhgW\UZh'hY]\ni \UVYb"'8]Yg'_ObbhY
]bgVYgcbXYfY'ZØf': fUi Yb'i bX'Y b[Y'A YbgW\Yb'j cb'J cfhY]\gY]b\z*
X]Y'cZh'j cb'hfUX]h]cbY\`Yb':]bUbngmghYa Yb'Ui g[YgW\`cggYb'g]bX"

5``YfX]b[g'[]Vh'Yg'Ui W\ < YfUi gZcfXYfi b[Yb'i bX'F]g]_Ybž'X]Y'a]h XYf'VfY]hYb'5_nYdhUbn'j cb'?fmdhck À\fi b[Yb'j YfVi bXYb'g]bX" 8 Uni '[Y\ÒfYb'XUg'DchYbn]U`'ZØf'Z]bUbn]Y``Y'6YhfØ[YfY]Ybž G]W\Yf\Y]hgVYXYb_Yb'i bX'X]Y'; YZU\fž'XUgg'?fmdhck À\fi b[Yb'ZØf]``Y[U`Y'5_h]j]hÀhYb'[Ybi hnh'k YfXYb"'8 U\Yf']gh'Y]bY'Ub[Ya YggYbY FY[i `Uh]cb'i bX'6]`Xi b['ØVYf'X]Y'F]g]_Yb'i bX'7\UbWYb'j cb ?fmdhck À\fi b[Yb'j cb'YbhgW\Y]XYbXYf'6YXYi hi b["

6 Ygi WX Yb 'G]Y 'i bg 'Ui Z. 'b!U["XY